$$\theta_1 = \sin^{-1}\left(\frac{\delta}{A}\right)$$

$$e = A\sin\omega t = A\sin\theta$$

$$f(e) = \begin{cases} mA\sin\theta & \text{if } 0 \le \theta \le \theta_1 \\ m\delta & \text{if } \theta_1 \le \theta \le \pi/2 \end{cases}$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(A\sin\theta)\sin\theta\, d\theta = \frac{4}{\pi}\int_0^{\pi/2} f(A\sin\theta)\sin\theta\, d\theta$$

note: because $f(A\sin\theta)$ is an odd function.

$$= \frac{4}{\pi}\int_0^{\theta_1} mA\sin^2\theta\, d\theta + \frac{4}{\pi}\int_{\theta_1} m\delta\sin\theta\, d\theta$$

$$= \frac{4}{\pi}\left[\frac{mA}{2}\int_0^{\theta_1}(1-\cos 2\theta)\, d\theta + m\delta(-\cos\theta)\Big|_{\theta_1}^{\pi/2}\right]$$

$$= \frac{2}{\pi}\left[mA\sin^{-1}\left(\frac{\delta}{A}\right) + m\delta\sqrt{1-\frac{\delta^2}{A^2}}\right]$$

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(A\sin\theta)\sin\theta\, d\theta = 0$$

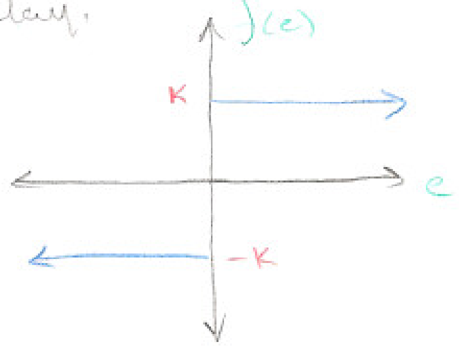
because it is odd function.

Finally we have

$$N(A) = \frac{a_1}{A} = \frac{2m}{\pi}\left[\sin^{-1}\left(\frac{\delta}{A}\right) + \frac{\delta}{A}\sqrt{\frac{A^2-\delta^2}{A^2}}\right]$$

note: for $A > \delta$, $A \le \delta \Rightarrow N = m$.

EX: Relay.



note: Same as last model but $\delta = 0$; $m\delta = K$

$\lim_{\delta \to 0} N(A)$

$m\delta = K$

$$N(A) = \frac{a_1}{A} = \frac{2K}{\pi}\left[\frac{1}{A}\underbrace{\frac{\sin^{-1}(\delta/A)}{\delta/A}}_{1} + \frac{1}{A}\underbrace{\sqrt{\frac{A^2+\delta^2}{A^2}}}_{1}\right]$$

$$= \frac{2K}{\pi}\left[\frac{1}{A} + \frac{1}{A}\right] = \frac{4K}{\pi A}$$

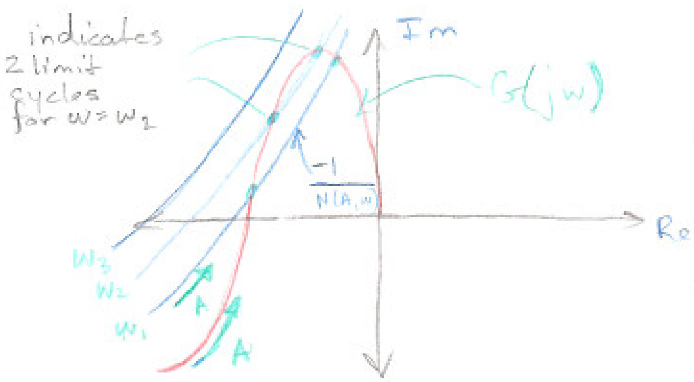
and we get a characteristic eqn

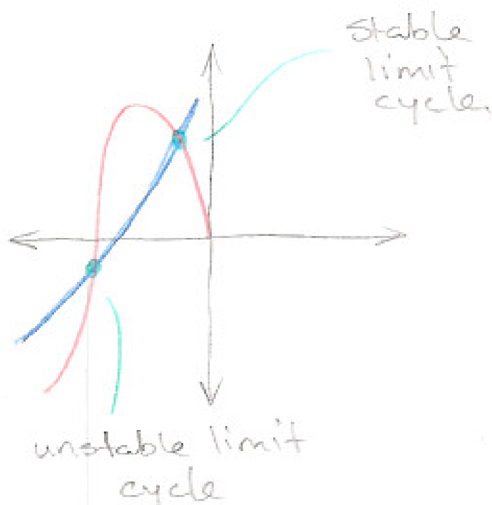
$$1 + G(j\omega)N(A,\omega) = 0$$

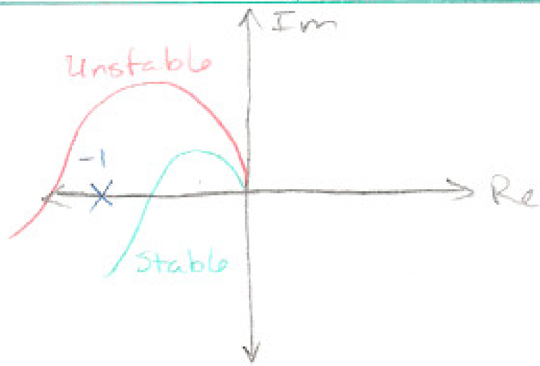
solve for ω and A

$$G(j\omega) = \frac{-1}{N(A,\omega)}$$

NYQUIST PLOT



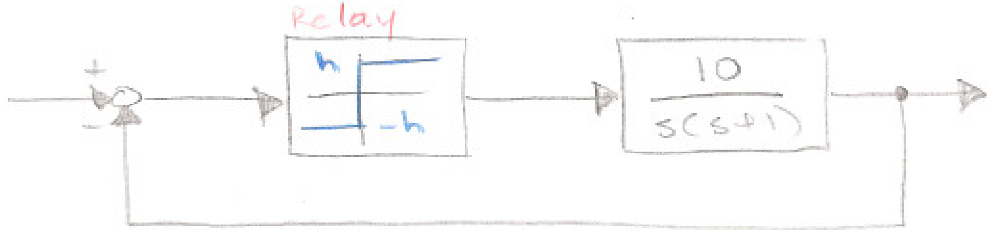




$$1 + G(j\omega) = 0$$

$$G(j\omega) = -1$$

EX:



$$N(A) = \frac{4h}{A\pi} \Rightarrow \frac{-1}{N(A)} = \frac{-\pi A}{4h}$$

$$G(j\omega) = \frac{10}{j\omega(j\omega+1)} = \frac{-10}{1+\omega^2} - j\frac{10}{\omega(1+\omega^2)}$$

$$G(j\omega) = \frac{-1}{N(A)}$$

$$\frac{-10}{1+\omega^2} - j\frac{10}{\omega(1+\omega^2)} = \frac{-\pi A}{4h}$$

NO LIMIT CYCLE

if $G(s) = \dfrac{10}{s(s+1)(s+2)}$

$$Re(G(j\omega)) = \frac{-30}{(1+\omega^2)(4+\omega^2)}$$

$$Im(G(j\omega)) = \frac{-10(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$G(j\omega) = \frac{-\pi A}{4h} \qquad Re(G(j\omega)) = \frac{-\pi A}{4h}$$

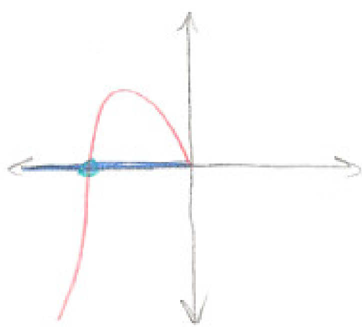
$$Im(G(j\omega)) = 0 \quad \therefore \omega = \sqrt{2}$$

$\therefore$ limit cycle is stable

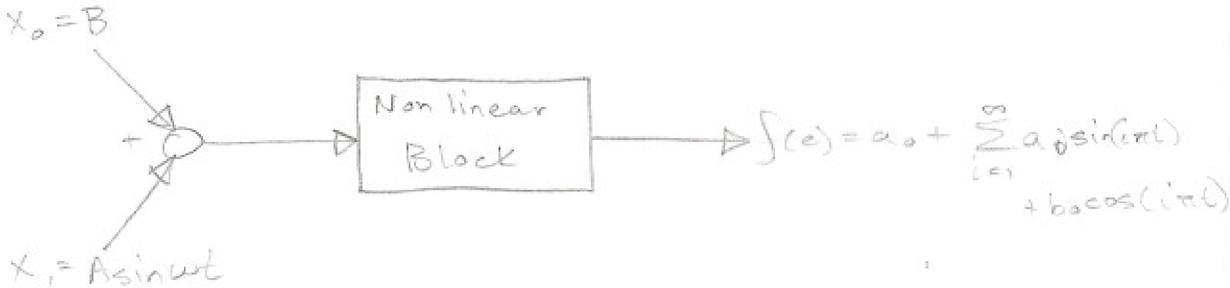
$$A = \frac{20h}{3\pi} = 2.122h$$

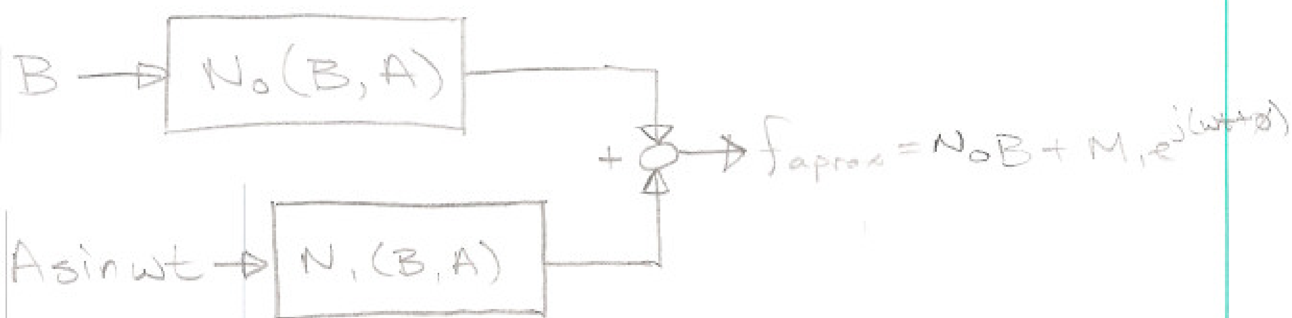
LIMIT CYCLE WHEN $\omega = \sqrt{2}$ AND $A = 2.122h$

PLOT:



SINUSOID AND DC BIAS DESCRIBING FUNC.





$$N_0 = \frac{a_0}{B} \qquad M_1 = \frac{\sqrt{a_1^2 + b_1^2}}{A} \qquad \phi_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

$$N_1 = M_1 \angle \phi_1$$